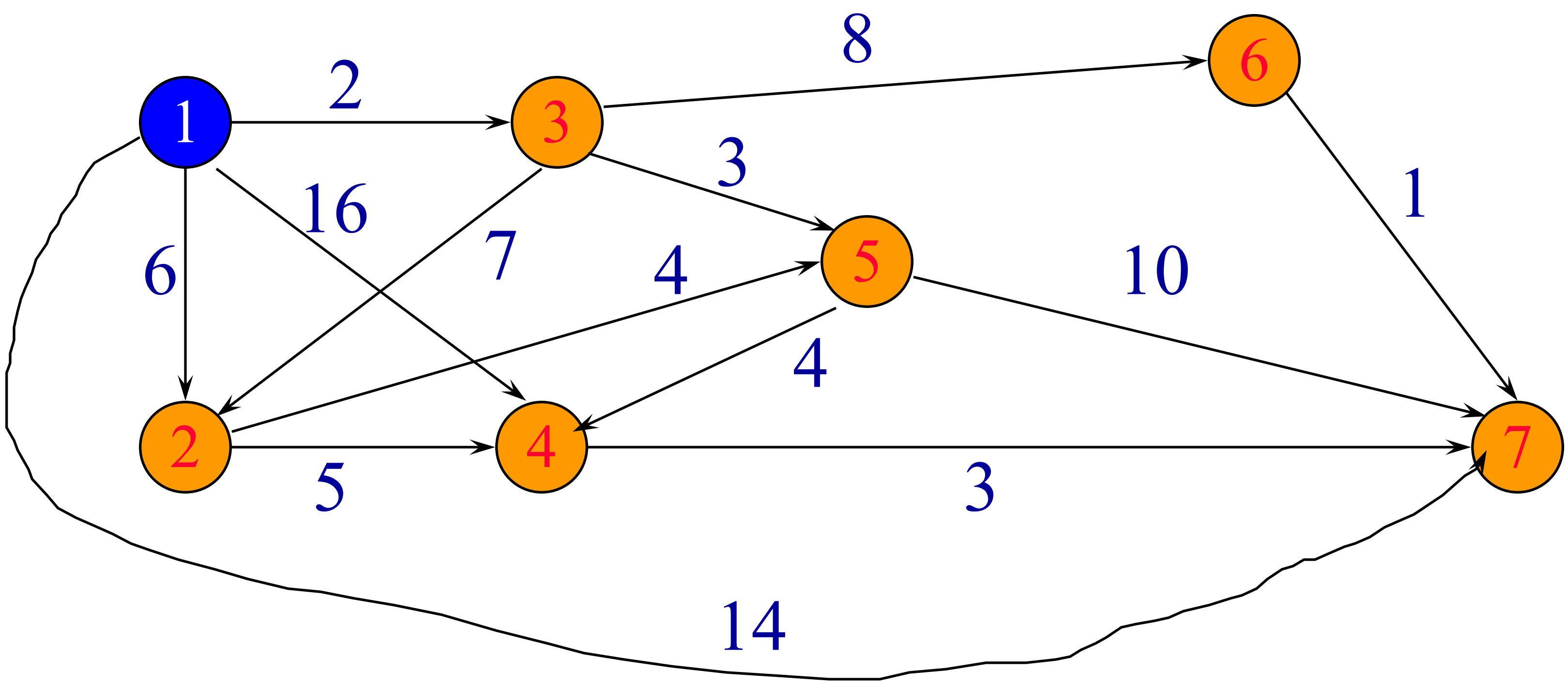
Fibonacci Heaps

	Actual	Amortized
Insert	O(1)	O(1)
Remove min (or max)	O(n)	O(log n)
Meld	O(1)	O(1)
Remove	O(n)	O(log n)
Decrease key (or increase)	O(n)	O(1)

Analysis

- FibonacciAnalysis.ppt
- Video
  - www.cise.ufl.edu/~sahni/cop5536; Internet Lectures; not registered
  - COP5536_FHA.rm

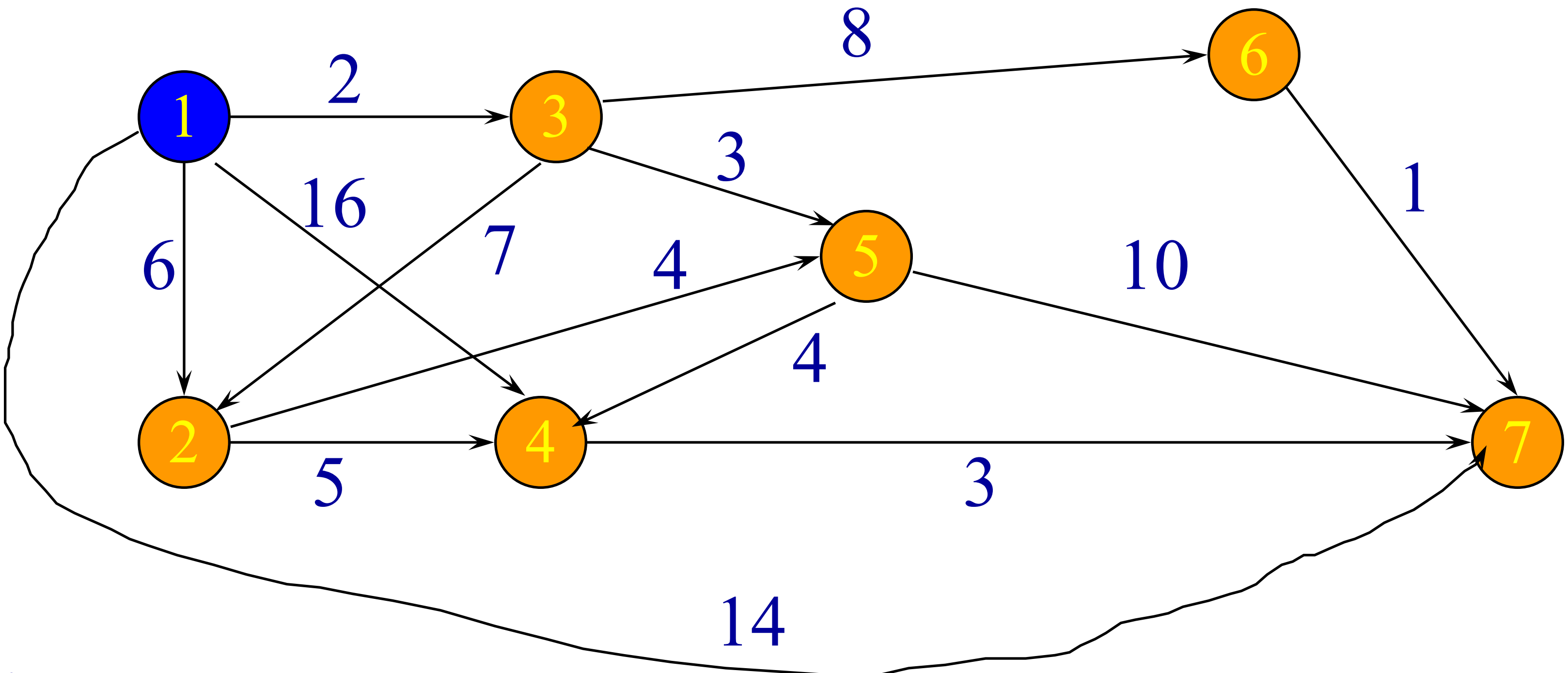
Single Source All Destinations Shortest Paths



Greedy Single Source All Destinations

- Known as Dijkstra's algorithm.
- Let d(i) be the length of a shortest one edge extension of an already generated shortest path, the one edge extension ends at vertex i.
- The next shortest path is to an as yet unreached vertex for which the d() value is least.
- After the next shortest path is generated, some d() values are updated (decreased).

Greedy Single Source All Destinations



Path	Length
1	0
1 → 3	2
1 → 3 → 5	5
1 → 2	6
1 → 3 → 5 → 4	9
1 → 3 → 6	10
1 → 3 → 6 → 7	11

Operations On d()

- Remove min.
  - Done O(n) times, where n is the number of vertices in the graph.
- Decrease d().
  - Done O(e) times, where e is the number of edges in the graph.
- Array.
  - $O(n^2)$ overall complexity.
- Min heap.
  - $O(n\log n + e\log n)$ overall complexity.
- Fibonacci heap.
  - $O(n\log n + e)$ overall complexity.

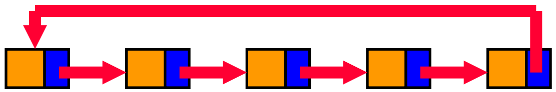
Prim's Min-Cost Spanning Tree Algorithm

- Array.
  - $O(n^2)$ overall complexity.
- Min heap.
  - $O(n\log n + e\log n)$ overall complexity.
- Fibonacci heap.
  - $O(n\log n + e)$ overall complexity.

Min Fibonacci Heap

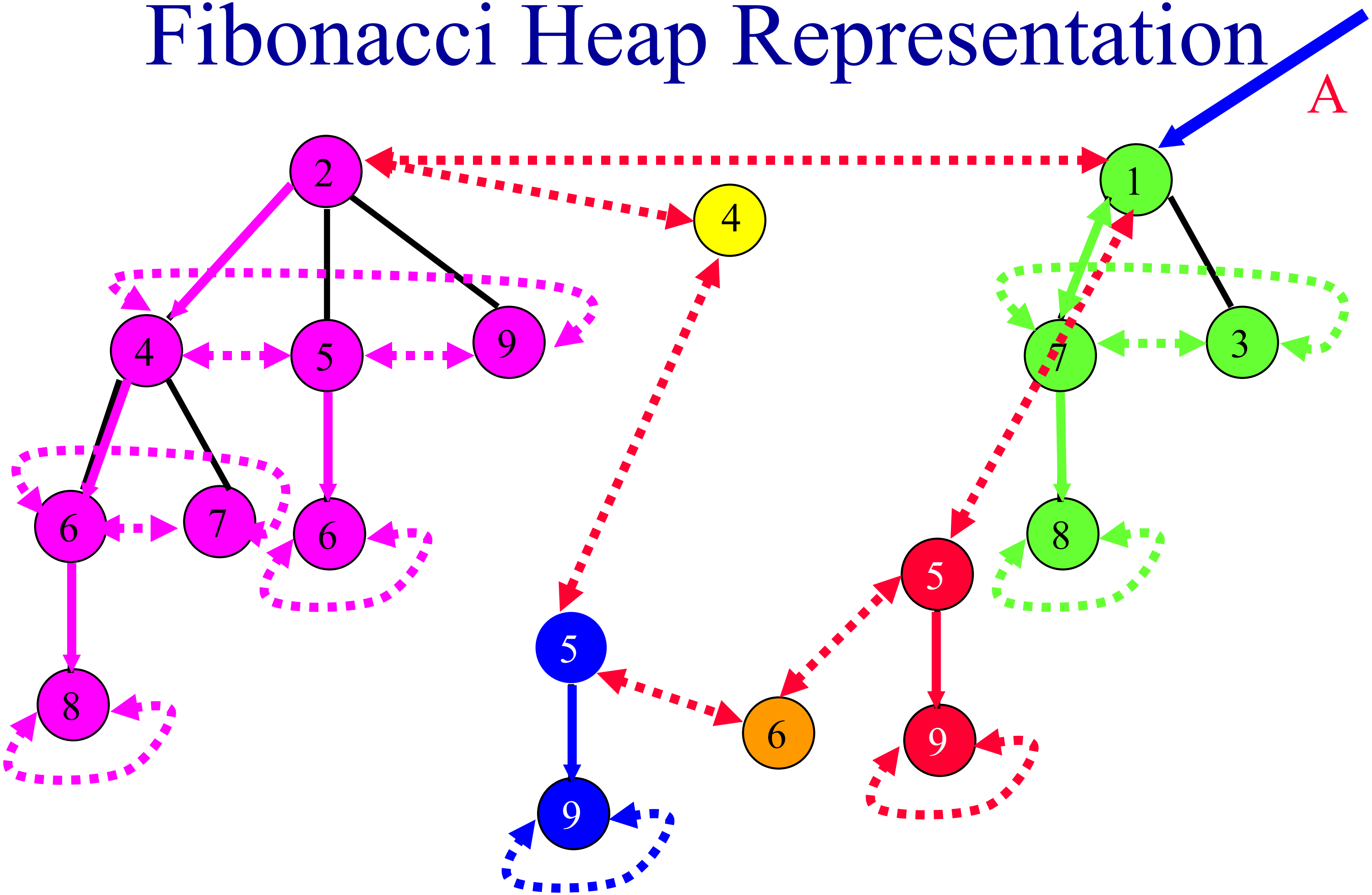
- Collection of min trees.
- The min trees need not be Binomial trees.

Node Structure



- Degree, Child, Data
- Left and Right Sibling
  - Used for circular doubly linked list of siblings.
- Parent
  - Pointer to parent node.
- ChildCut
  - True if node has lost a child since it became a child of its current parent.
  - Set to false by remove min, which is the only operation that makes one node a child of another.
  - Undefined for a root node.

Fibonacci Heap Representation



- Degree, Parent and ChildCut fields not shown.

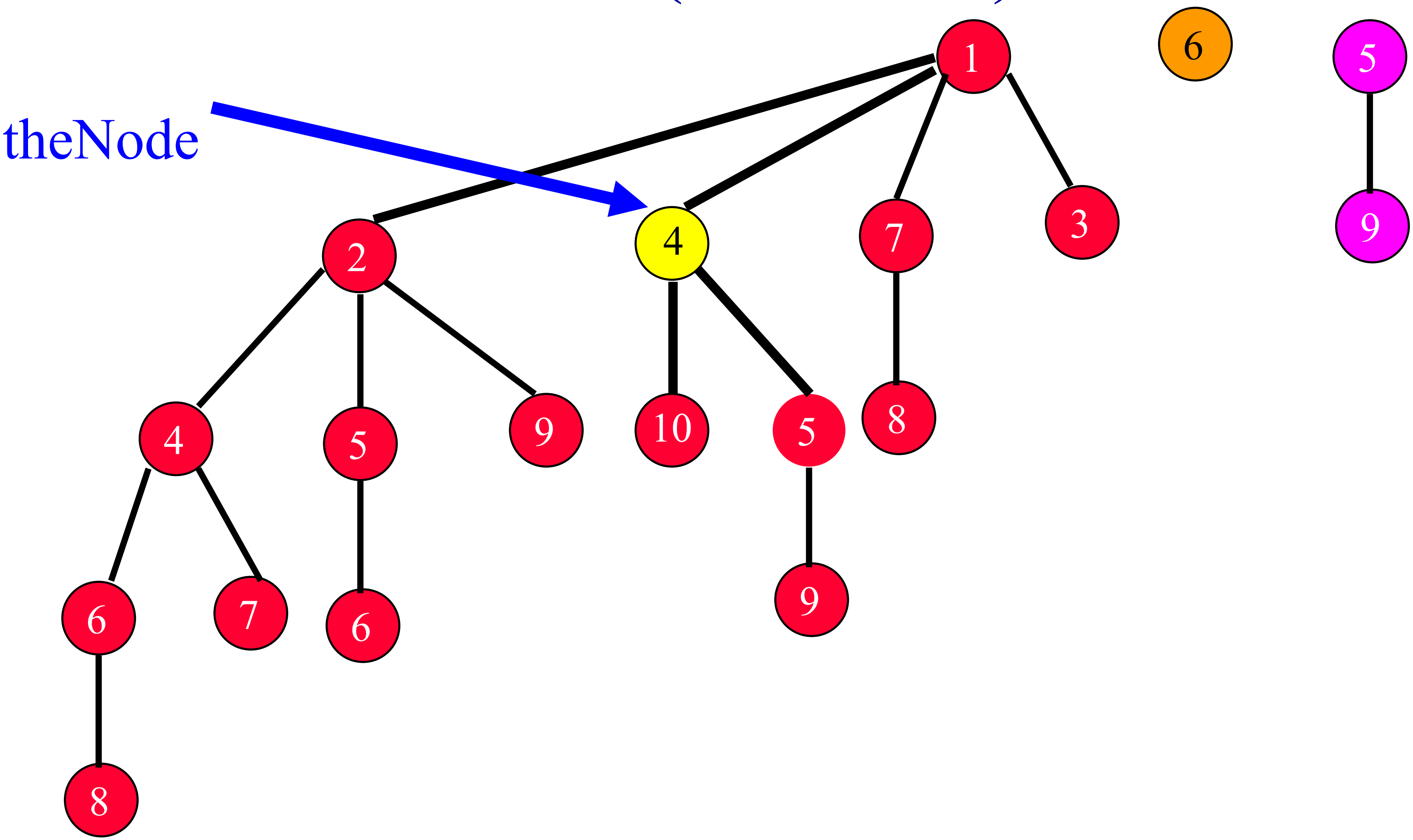
Remove(theNode)

- theNode points to the Fibonacci heap node that contains the element that is to be removed.
- theNode points to min element => do a remove min.
  - In this case, complexity is the same as that for remove min.

Remove(theNode)

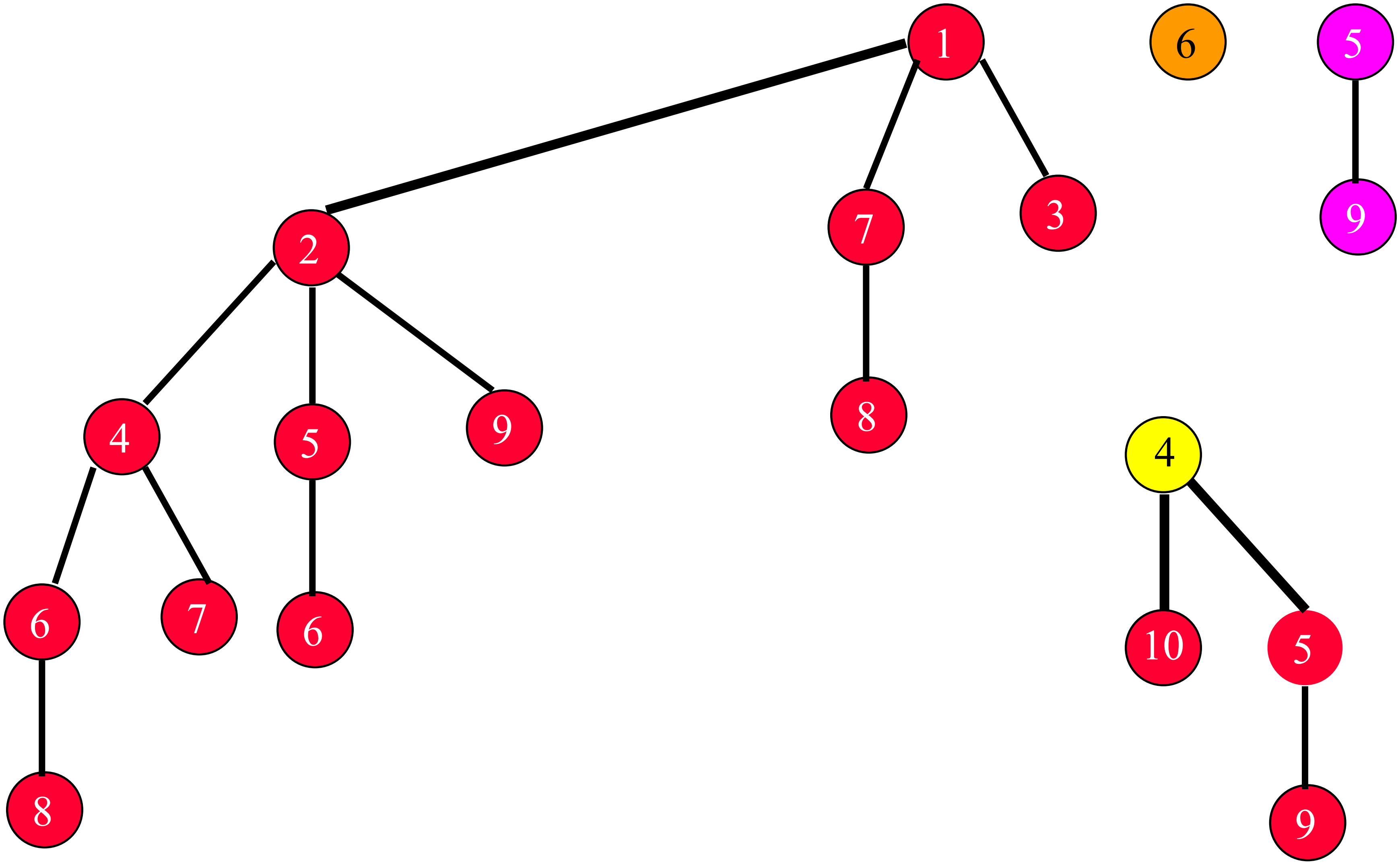
- theNode points to an element other than the min element.
  - Remove theNode from its doubly linked sibling list.
  - Change parent's child pointer if necessary.
  - Set parent field of theNode's children to null.
  - Combine top-level list and children list of theNode; do not pairwise combine equal degree trees.
  - Free theNode.
- In this case, actual complexity is O(log n) (assuming theNode has O(log n) children).

Remove(theNode)



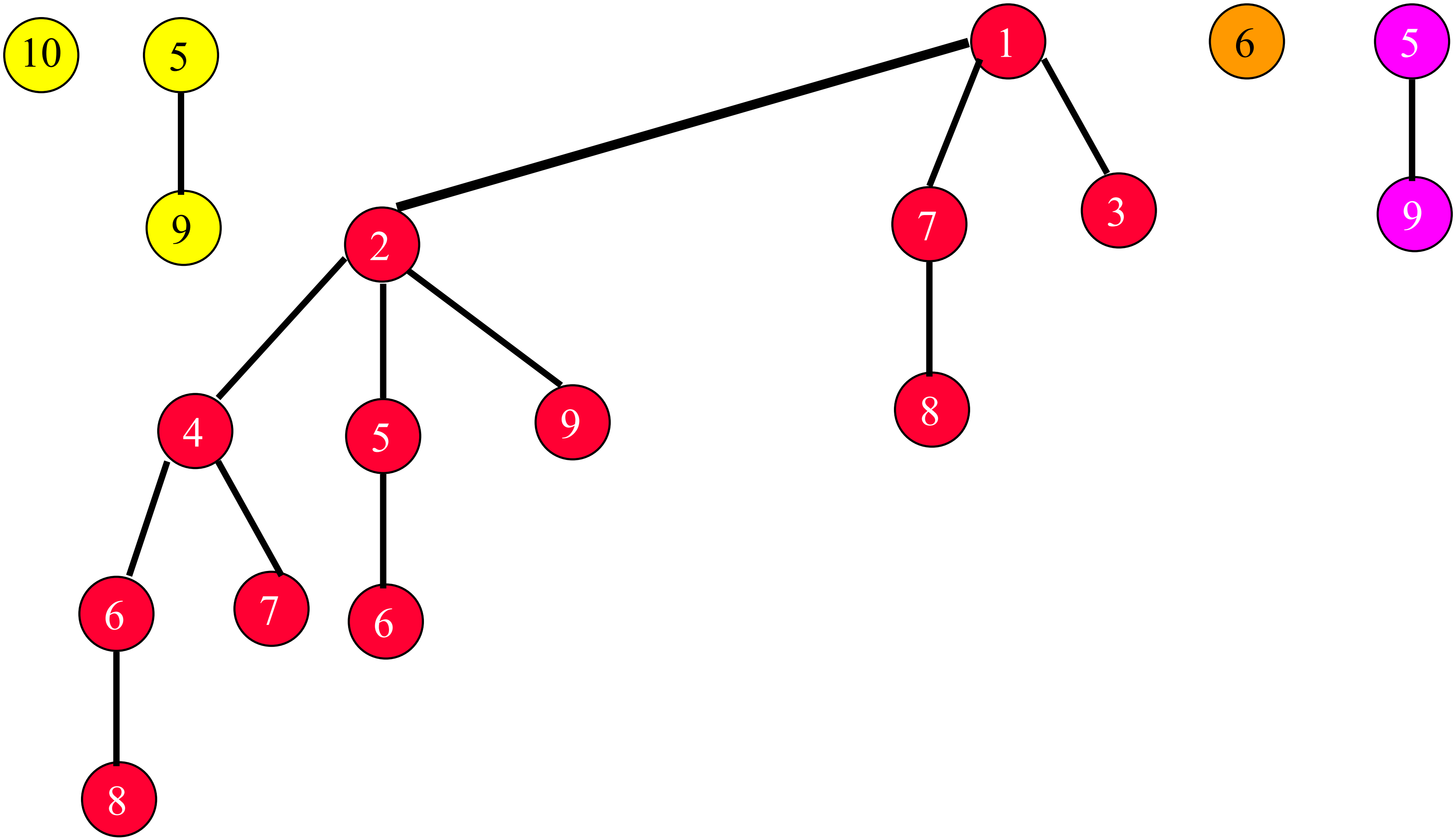
Remove theNode from its doubly linked sibling list.

Remove(theNode)

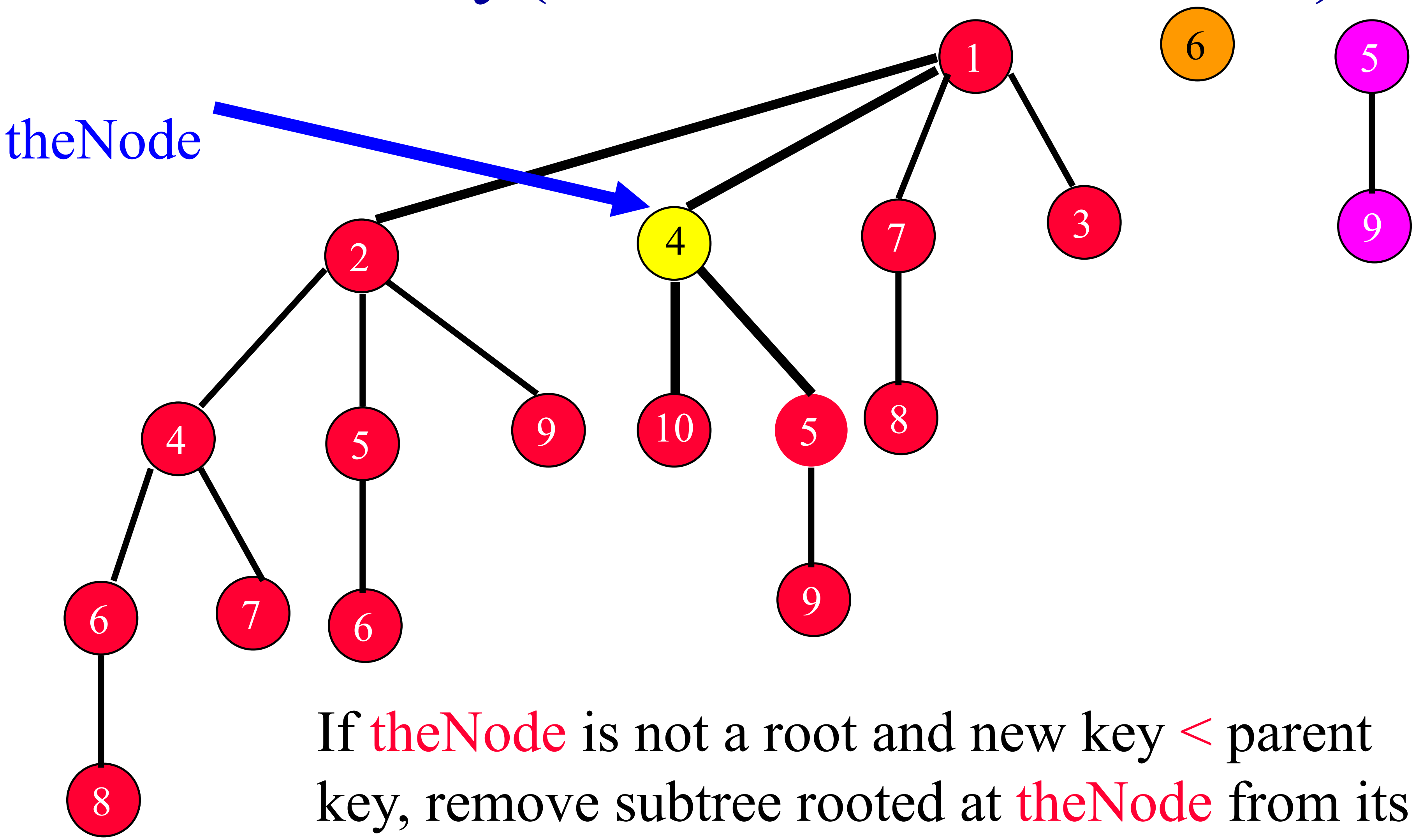


Combine top-level list and children of theNode setting parent pointers of the children of theNode to null.

Remove(theNode)



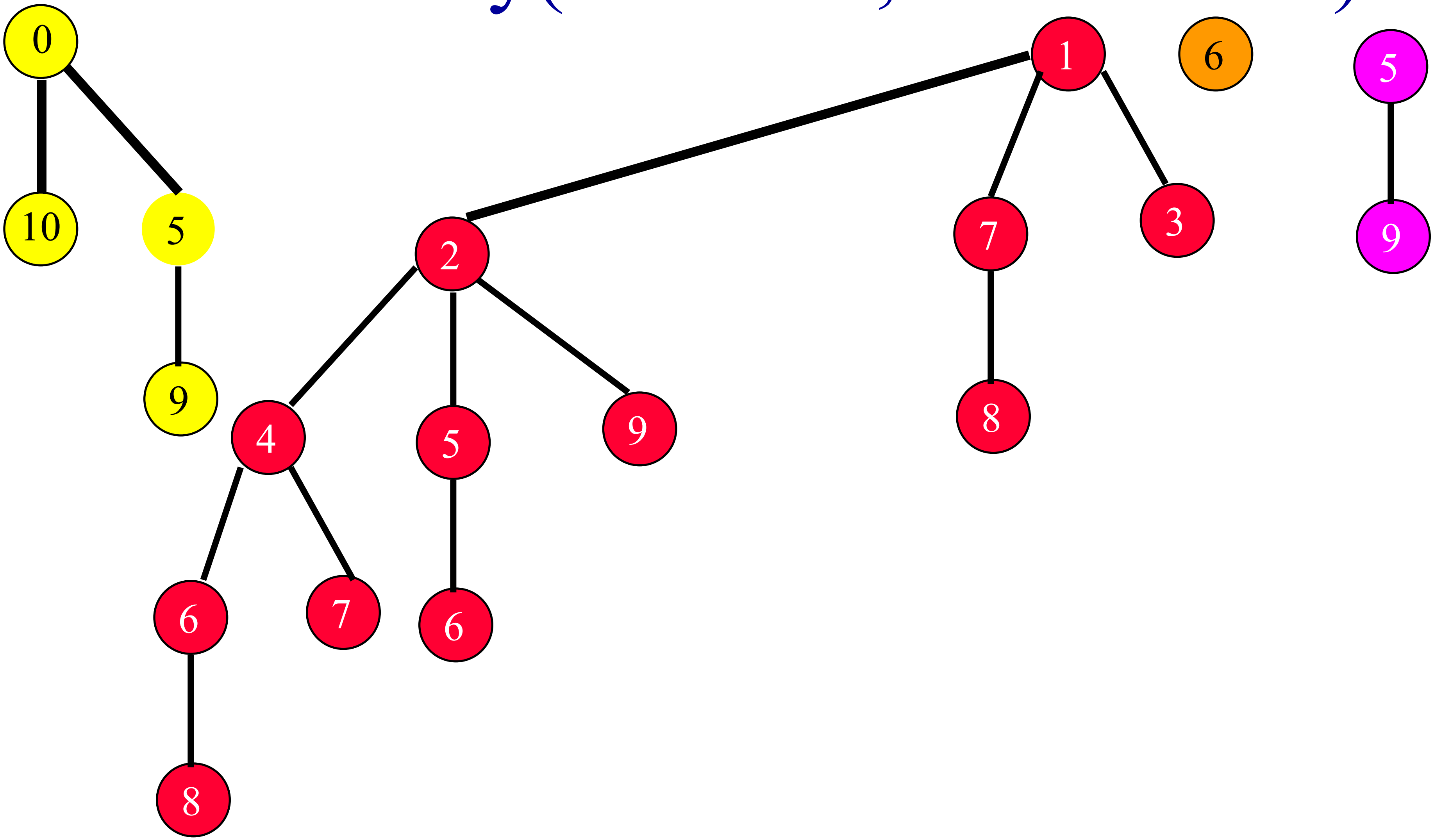
DecreaseKey(theNode, theAmount)



If theNode is not a root and new key < parent key, remove subtree rooted at theNode from its doubly linked sibling list.

Insert into top-level list.

DecreaseKey(theNode, theAmount)

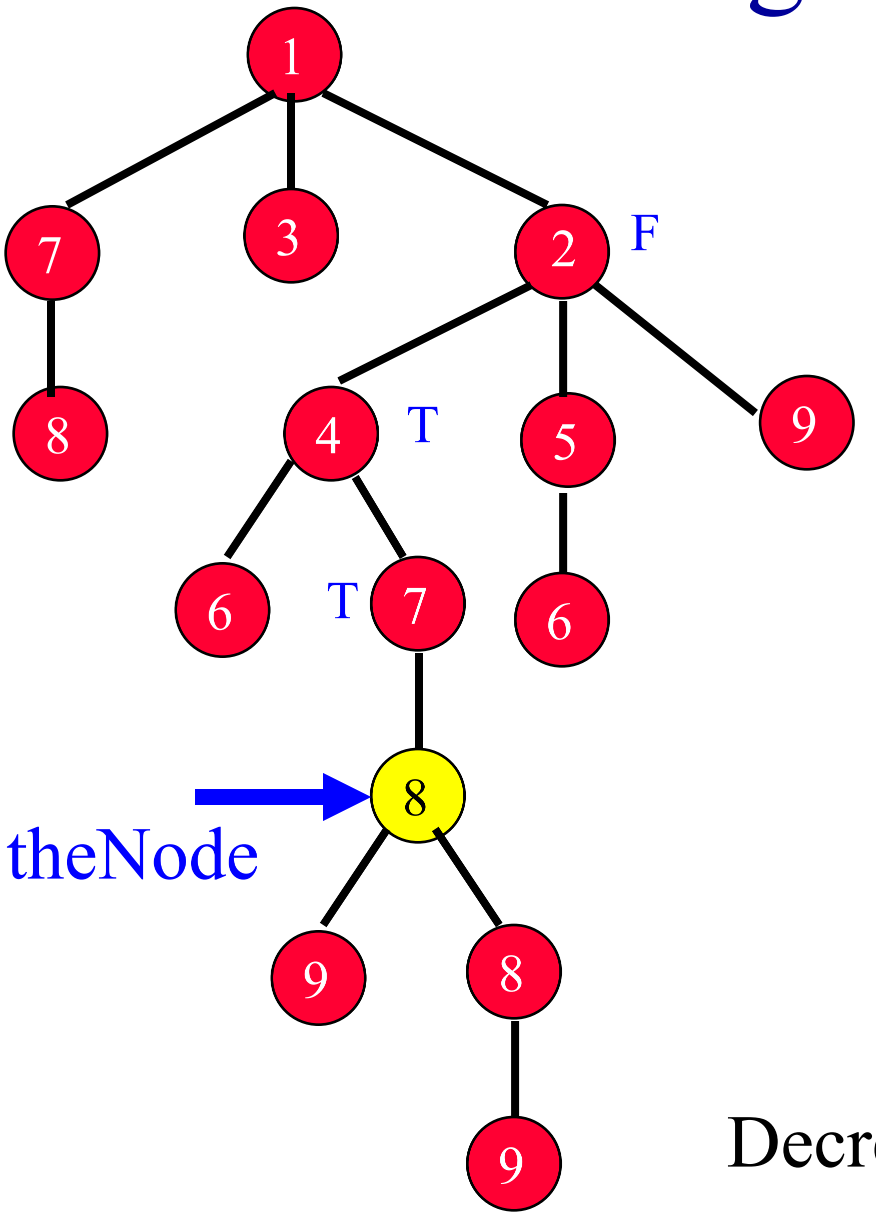


Update heap pointer if necessary

Cascading Cut

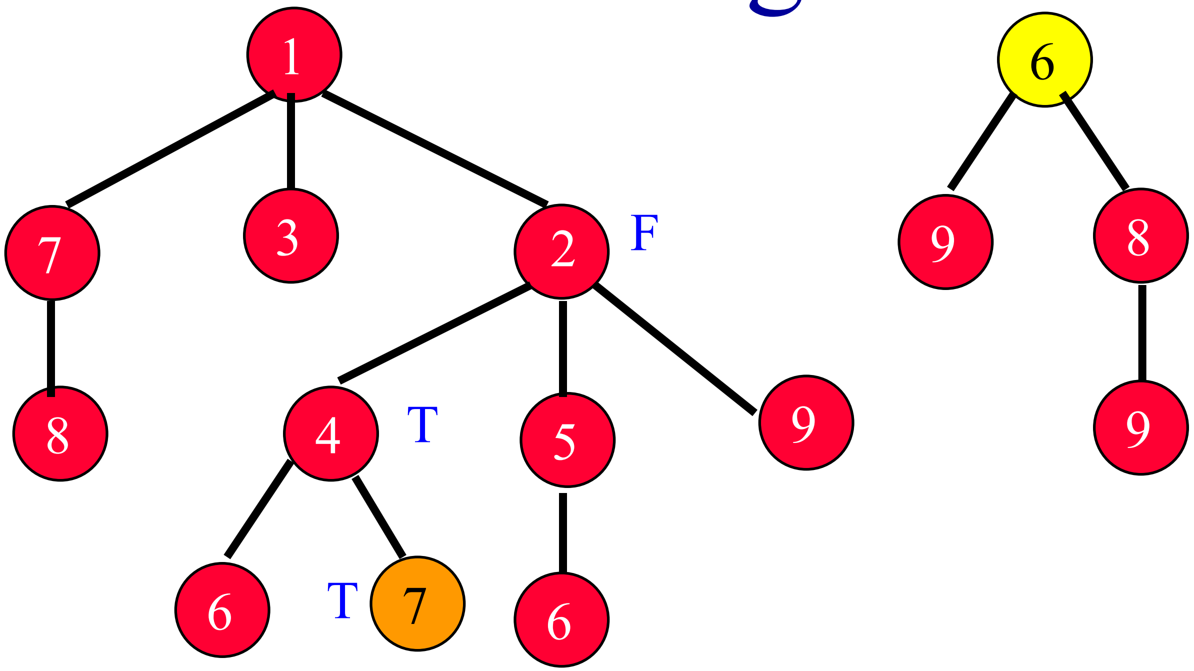
- When theNode is cut out of its sibling list in a remove or decrease key operation, follow path from parent of theNode to the root.
- Encountered nodes (other than root) with ChildCut = true are cut from their sibling lists and inserted into top-level list.
- Stop at first node with ChildCut = false.
- For this node, set ChildCut = true.

Cascading Cut Example



Decrease key by 2.

Cascading Cut Example



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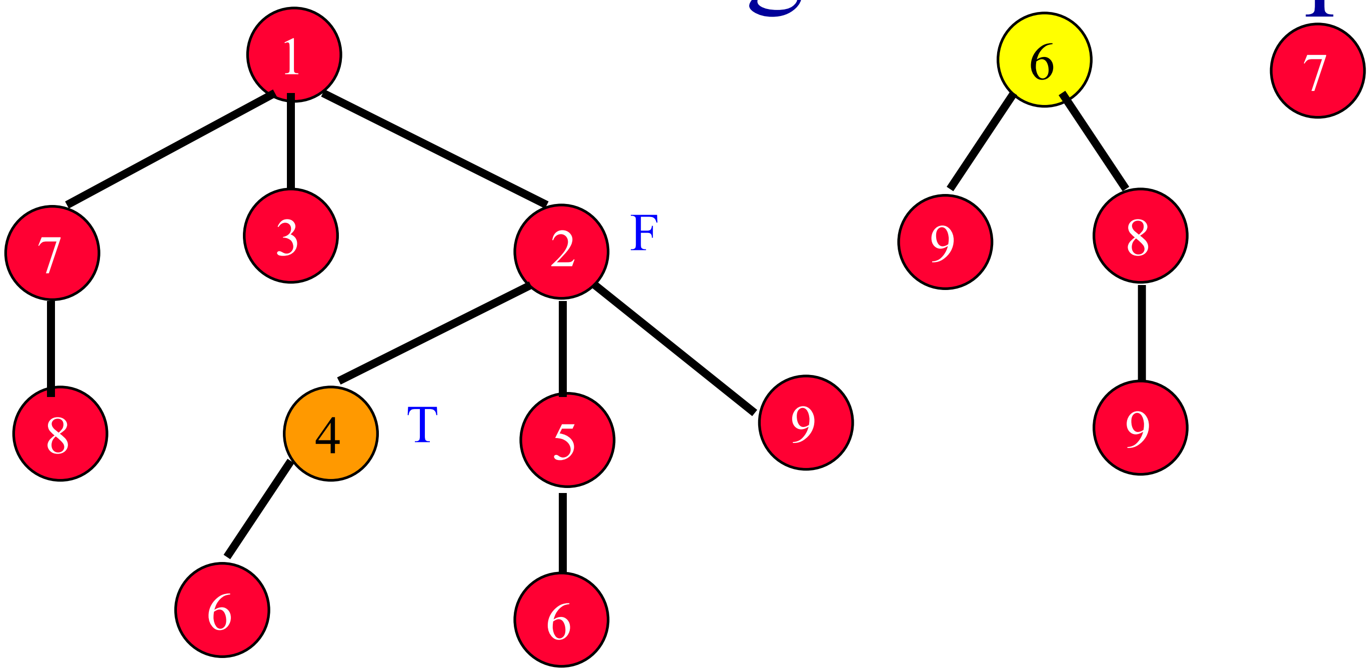

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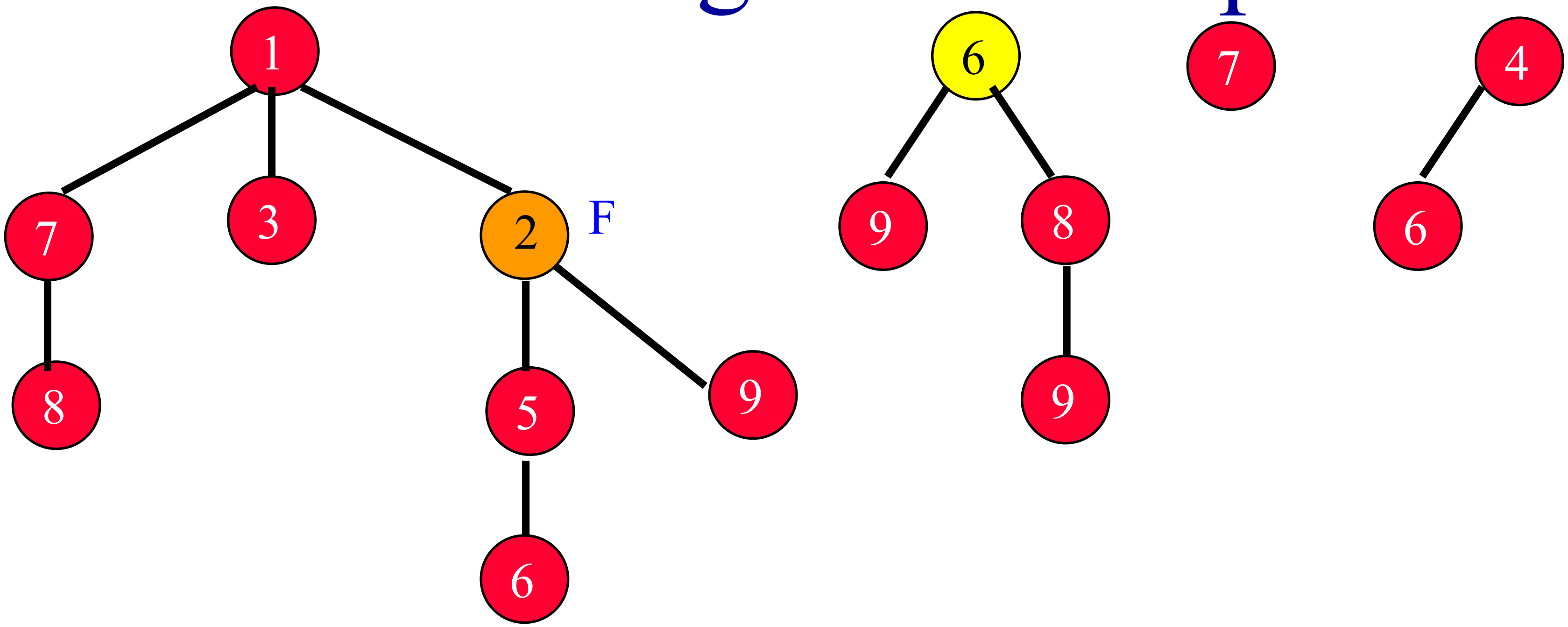
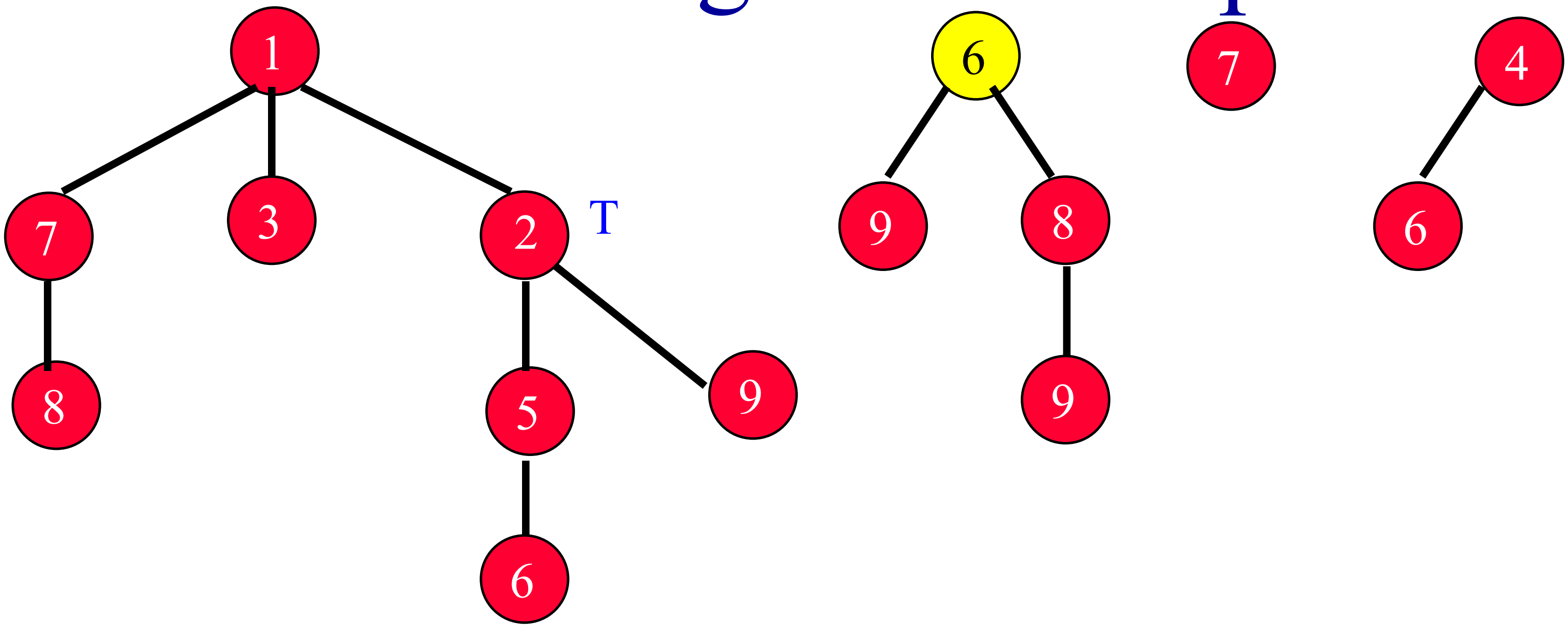


Cascading Cut Example



Actual complexity of cascading cut is O(h) = O(n).