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The maintenance of cooperation in multiplex networks with limited and partible resources of agents

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HIGHLIGHTS

- Agents play iterated public goods game in multiplex networks with limited and partible resources.
- High degree diversity in one layer promotes cooperation in multiplex networks.
- Degree differences between conjoint nodes encourage cooperative behaviors.
- A greedy-first mechanism that facilitates the emergence of cooperation is proposed.

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ABSTRACT

In this paper, we try to explain the maintenance of cooperation in multiplex networks with limited and partible resources of agents: defection brings larger short-term benefit and cooperative agents may become defective because of the unaffordable costs of cooperative behaviors that are performed in multiple layers simultaneously. Recent studies have identified the positive effects of multiple layers on evolutionary cooperation but generally overlook the maximum costs of agents in these synchronous games. By utilizing network effects and designing evolutionary mechanisms, cooperative behaviors become prevailing in public goods games, and agents can allocate personal resources across multiple layers. First, we generalize degree diversity into multiplex networks to improve the prospect for cooperation. Second, to prevent agents allocating all the resources into one layer, a greedy-first mechanism is proposed, in which agents prefer to add additional investments in the higher-payoff layer. It is found that greedy-first agents can perform cooperative behaviors in multiplex networks when one layer is scale-free network and degree differences between conjoint nodes increase. Our work may help to explain the emergence of cooperation in the absence of individual reputation and punishment mechanisms.

1. Introduction

One of the most critical problems in evolutionary dynamics is to find the reason for the maintenance of cooperative behaviors when defective behaviors can lead to larger short-term benefits [1,2]. Recently, the empirical resolution of different linking types has been improved [3,4], and the real social systems can be described as a superposition of several complex social networks, which is commonly named as multiplex networks [5–9]. Many studies have generalized evolutionary games into multiplex networks and accounted for the positive effects of multiplex structure on evolutionary

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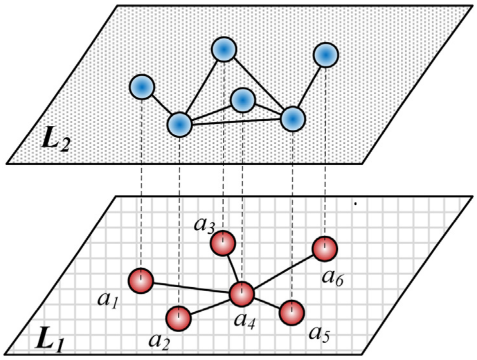

Fig. 1. Illustration of multiplex networks.

cooperation [10–17]. Some studies also consider several ways of linking different networks, which focus on the effects of the interconnections among different layers [18,19]. In conjoint layers of multiplex networks, each agent is usually expected to cooperate in multiple games simultaneously. However, few works have considered the "payment capability" which indicates the maximum disposable resources (costs) of the agent and the restriction between costs of multiple games. The limited resources of agents are common restrictions in many multi-agent systems [20,21] and real social systems [3].

This raises a fundamental problem for evolutionary cooperation in multiplex networks: *How do the limited resources of agents affect the positive effects of multiplex structure on cooperation evolution*? Because of larger short-term benefits, defective behaviors can be easily imitated. Therefore, can cooperative behaviors finally disappear if some agents have to defect in partial interactions due to the unaffordable costs of cooperation in all the interactions?

In this case, there are two defective temptations that can inhibit cooperative behaviors in a certain layer. The first temptation is the higher temporal payoffs of defective neighbors, which is inherent in the maintenance of evolutionary cooperation [1,2,22]. Most present studies focus on network reciprocity of multiplex structure which means that cooperative behaviors can exist and diffuse if agents can aggregate into cooperative clusters [23]. The second temptation originates in different payoffs of allocated resources in multiple layers, which may lead to the disappearance of cooperation in the lower-payoff layers. To ensure the non-zero payoff in some layers, agents should contribute parts of resources in multiple layers or adopt biased allocation strategies in different layers. Therefore, it is necessary to design efficient mechanisms to induce proper strategies of resource allocation in multiple layers.

Our work utilizes the effects of network structure and designs evolutionary mechanisms to ensure the cooperative behaviors in iterated public goods games (PGGs) with limited and partible resources of agents. First, to defeat the temptation of defective neighbors in a certain layer, we generalize degree diversity [24] into multiplex networks. Second, we design a greedy-first evolutionary mechanism to prevent the disappearance of cooperation in the lower-payoff layer. The greedy-first agents prefer to imitate the allocation strategies of neighbors in the higher-payoff layer. Correspondingly, a generous-first mechanism, that means agents tend to improve the payoffs in the lower-payoff layer, is also proposed to highlight the effect of greedy-first mechanism.

Specifically, we give specific conditions to analyze how degree diversity and greedy-first mechanism can promote cooperative behaviors in multiplex networks. It is worth noting that Santos et al. [24] firstly introduce the degree diversity in PGGs, which is played in the single-layer network with binary contributions of resources. In this paper, we further extend the close relationship between degree diversity and the maintenance of cooperation from two additional perspectives. First, cooperative behaviors in multiplex networks can be promoted if degrees of nodes in one layer are highly diverse. Second, evolutionary cooperation can also be facilitated by the differences between degrees of conjoint nodes in multiple layers. Moreover, greedy-first mechanism utilizes the human instinct of searching higher benefits and may help to understand the emergence of cooperation. In particular, our work suggests that cooperative behaviors can be controlled by varying the degree diversity in multiplex networks.

2. Model statement

2.1. Topology and agent

Multiplex networks consist of two or more layers. In our model, we only consider the case of duplex networks which have two layers denoted by L_1 and L_2, and each layer indicates one type of relationship between humans. Parallel graphs, which are conjoint by cross-layer paths, are usually used to represent multiplex networks [8,9]. As shown in Fig. 1, each agent represents a pair of conjoint nodes and is linked with different sets of agents in the duplex layers. For agent i, the notation $N_i^{L_1}$ ($N_i^{L_2}$) denotes the set of i and its neighbors, and $k_i^{L_1}$ ($k_i^{L_2}$) denotes the degree of i in L_1 (L_2), respectively.

Agent i possesses a constant amount of resources R_e. For the sake of simplicity, R_e is identical for all the agents and set as one unit. The notations $c_i^{L_1}$ and $c_i^{L_2}$ indicate the percentage of resources that are allocated in each layer. Thus, $c_i^{L_1} \geq 0$,

$c_i^{L_2} \geq 0$, and $c_i^{L_1} + c_i^{L_2} \leq 1$. The above three inequations show the interdependent costs of multiple games in the duplex layers, which are the fundamental limitations of evolutionary dynamics.

To reduce parameter spaces of allocation strategies, $c_i^{L_1}$ and $c_i^{L_2}$ are discrete and the multiple of the variation 0.1, such as $c_i^{L_1} = 0.3$, $c_i^{L_2} = 0.7$ or $c_i^{L_1} = 0.5$, $c_i^{L_2} = 0.2$. Therefore, there are 66 combinations of allocation strategies. Before evolutionary dynamics begins, initial strategies of agents are randomly selected from these combinations. In our model, the agent in multiplex networks is defective if $c_i^{L_1} = c_i^{L_2} = 0$, and i is more cooperative than j in L_1 if $c_i^{L_1} > c_j^{L_1}$. If $c_i^{L_1} + c_i^{L_2} > 0$ and $c_i^{L_1} \times c_i^{L_2} = 0$, agent i is partially cooperative (biased cooperation).

2.2. Public goods game

In order to clearly describe iterated public goods games in multiplex networks, it is necessary to introduce the general principle of PGG. In the classical public goods game, every cooperator contributes a part of resource c, which is nonzero and not larger than the total amount of resources; and defectors contribute nothing to the public pot [25–28]. The total contribution is multiplied by an enhancement r, which is generally larger than one and less than the number of players. The result is equally distributed among all the players. In the classical public goods game, all players are unstructured (or well-mixed). Santos et al. [24] consider the connectivity of human and extend the PGG into social networks: each agent receives contributions of neighbors and shares the enhanced benefit between itself and neighbors. For an agent i with degree k, i divides its contributed resources into $k + 1$ parts equally and distributes them to $k + 1$ PGGs which center on i and its neighbors. In the PGG proposed by Santos et al. [24], cooperators contribute the total resources at once, where only the single-layer social network is analyzed.

We generalize the PGG proposed by Santos et al. into multiplex network with the consideration of the limited and partible resources. In each layer, the agent distributes the resources between itself and its neighbors and participates in all PGGs [24]. The payoff of the agent in one layer depends on the number of the neighbors in the same layer, the total amount of resources that it receives and the enhancement factor. In each PGG, the payoff of the agent is calculated for the following processes: first, the total amount of resources received by the central agent of this PGG is multiplied by the enhancement factor; and then, the result subtracts the amount of resources that the agent contributed to this PGG. Therefore, each agent i contributes $c_i^{L_1} / \left(k_i^{L_1} + 1\right)$ resources in each PGG in L_1 and the payoff $P_i^{L_1}$ of i in L_1 is given by

$$P_i^{L_1} = \sum_{j \in N_i^{L_1}} \left[\left(\frac{r}{k_j^{L_1} + 1} \sum_{x \in N_j^{L_1}} \frac{c_x^{L_1}}{k_x^{L_1} + 1} \right) - \frac{c_i^{L_1}}{k_i^{L_1} + 1} \right] \tag{1}$$

where the payoffs of i in $k_i^{L_1} + 1$ PGGs in L_1 are accumulated. The payoff $P_i^{L_2}$ is calculated by the same process. We also assume that the enhancement factors are identical in the duplex layers of multiplex networks. For the convenience of readers, the major variables are listed in Table 1.

2.3. Greedy-first mechanism

Evolutionary mechanism determines the basic rule which governs the agent to update the strategy based on the imitation of the neighbor's strategy which can bring more benefit [24,12]. The strategies of agents are asynchronously updated. After each game round, one agent is randomly selected to update the behavior strategy. Because of the limitation of connectivity, the agent can only perceive the partial strategies of neighbors in one layer instead of the entire allocation strategies in multiple layers.

At first, the randomly selected agent i has to choose one layer from multiplex networks, and then imitates the allocation strategy of a neighbor. We design a greedy-first mechanism, in which agent i prefers to update the allocation strategy in the higher-payoff layer according to a probability determined by the Fermi function

$$Pro_i\left(L_1 \,|L_2\right) = \frac{1}{1 + \exp\left[\left(P_i^{L_1} - P_i^{L_2}\right)/K\right]}. \tag{2}$$

The factor $K = 0.1$ quantifies the uncertainty associated with the strategy adoption process [12,13]. Agent i decides to update L_2 with $Pro_i(L_1|L_2)$ and choose L_1 with $1 - Pro_i(L_1|L_2)$. Then, supposing that L_1 is selected, i adopts the allocation strategy of a random neighbor j in L_1 with the probability $Pro\left(c_j^{L_1} \rightarrow c_i^{L_1}\right)$ also determined by the Fermi function

$$Pro\left(c_j^{L_1} \rightarrow c_i^{L_1}\right) = \frac{1}{1 + \exp\left[\left(P_i^{L_1} - P_j^{L_1}\right)/K\right]}. \tag{3}$$

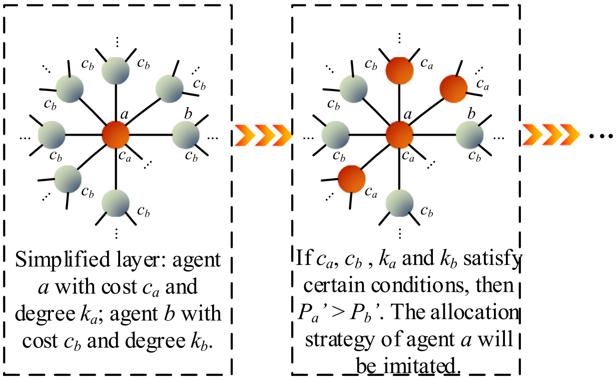

Fig. 2. Illustration of simplified layer with two kinds of agents.

Table 1
Major notation used in this paper.

Symbol	Explanation
$N_i^{L_1}$ ($N_i^{L_2}$)	The set of i and its neighbors in L_1 (L_2)
$k_i^{L_1}$ ($k_i^{L_2}$)	The degree of i in L_1 (L_2)
R_e	The constant amount of resources
$c_i^{L_1}$ ($c_i^{L_2}$)	The percentage of resources that i allocates in L_1 (L_2)
r	The enhancement factor in PGGs
$P_i^{L_1}$ ($P_i^{L_2}$)	The payoff of i in L_1(L_2)

If i adopts the allocation strategy of j in L_1 and the sum of the resources allocated in two layers is larger than R_e, the percentage of resources allocated in L_2 needs to be changed: $c_i^{L_2} = R_e - c_i^{L_1}$. On the other hand, $c_i^{L_1}$ remains unchanged with the probability $1 - Pro\left(c_j^{L_1} \rightarrow c_i^{L_1}\right)$.

The greedy-first mechanism indicates the nature of searching higher benefit when the individual is in the face of multiple choices. To highlight the effect of greedy-first mechanism, a generous-first mechanism is also proposed: i decides to update L_1 with $Pro_i(L_1|L_2)$ and choose L_2 with $1 - Pro_i(L_1|L_2)$. This mechanism indicates that the agent wants to increase the benefit in the lower-payoff layer by imitating the allocation strategy of an agent with higher payoff.

3. Analysis of evolutionary dynamics

It is necessary to theoretically analyze evolutionary dynamics with a given condition. In this section, we first prove that greedy-first mechanism is superior to generous-first mechanism. Second, we show that degree diversity is important to promote cooperative behaviors in multiplex networks in the case that the resources are limited and partible.

Because it is very difficult and complicate to demonstrate the detail of the evolutionary PGG dynamics on complex networks, it is common to resort to a simple and typical case model—a single agent a is in the cluster of other homogeneous agents b without any short and closed loops. The degree of agent b is k_b (one agent a and $k_b - 1$ agent b), and agent a has $k_a b$-neighbors. A brief illustration is shown in Fig. 2. Agent a contributes c_a to the network while agent b contributes c_b. Because most agents hold the allocation strategy c_b, it is very disadvantageous for agent a to spread the allocation strategy c_a unless the payoff of agent a is considerably larger than the one of agent b. According to the rule of PGGs described in Section 2.2, we discuss how the cooperative strategy of agent a can exist in the local environment.

In this scenario, the payoffs of agent a and b are given by

$$\begin{aligned} P_a &= \frac{r}{k_a+1}\left(\frac{c_a}{k_a+1}+\frac{k_a c_b}{k_a+1}\right)+\frac{r k_a}{k_b+1}\left(\frac{c_a}{k_a+1}+\frac{k_b c_b}{k_b+1}\right)-c_a \\ P_b &= \frac{r}{k_a+1}\left(\frac{c_a}{k_a+1}+\frac{k_a c_b}{k_a+1}\right)+\frac{r}{k_b+1}\left(\frac{c_a}{k_a+1}+\frac{k_b c_b}{k_b+1}\right)+\frac{(k_b-1)\, r c_b}{k_b+1}-c_b. \end{aligned} \tag{4}$$

Based on the above expressions, the allocation strategy of agent a stays more stable if the payoff of agent a is larger than that of agent b. It can be calculated that

$$\frac{\partial\left(P_a-P_b\right)}{\partial k_a} = r c_b + \frac{r c_a}{\left(k_a+1\right)^2} > r c_b > 0. \tag{5}$$

Therefore, in order to make sure that $P_a > P_b$, a larger degree k_a is preferred. In particular, if $k_a = k_b = k$, the difference between the two payoffs is calculated as

$$P_a - P_b \Big|_{k_a=k_b=k} = \frac{c_b - c_a}{k_b + 1}\left(k + 1 - r\frac{k-1}{k+1}\right). \tag{6}$$

In this situation, agent a can only contribute less resources than agent b to maintain a high payoff. Meanwhile, if $k_a > k_b$, agent a may exploit the advantages of high degree: the payoff of agent a increases with a larger degree k_a. Then, the allocation strategy of agent a may remain unchanged even with a higher contribution of resources and agent b may imitate the strategy of agent a. Consequently, the total amount of contributed resources in the network increases. In other words, agents can adopt a more cooperative strategy with the higher probability if the diversity in degree distribution increases. However, the degrees between nodes are nearly the same ($k_a - k_b \approx 0$) in homogeneous networks, such as Erdős–Rényi random network [29] and small-world network [30]. As a result, the agent contributes fewer and fewer amount of resources because only more defective behavior leads to the higher payoff compared with linking neighbors.

According to the relation (5), the payoff of agent a with the cooperative strategy becomes larger than the one of agent b if $k_a - k_b$ and r further increase. Therefore, the hub nodes in the scale-free network are important to evolutionary cooperation. In the local multiple PGGs, cooperative hubs play important roles, and defective strategy is restrained among agents with low degrees. Thus, diversity helps to promote cooperation in a single layer.

It is worth noting that Santos et al. [24] firstly introduce the degree diversity into iterated public goods games with the binary contribution of resources ($c = 0$ or 1). Our work extends the model and proves the importance of degree diversity if resources are limited and partible. Based on the above analyses, we show that degree diversity of a single layer is important to the entire multiple networks.

On the other hand, the payoff of the agent in a certain layer is lower if the contributions of neighbors are lower. It is worth noting that the agent adopts the allocation strategy of one of its neighbors in the layer if the agent successfully updates the strategy. Then, it can be calculated that

$$\begin{aligned} \frac{\partial P_a}{\partial c_b} &= \frac{rk_a}{k_b+1}\left(1 + \frac{1}{k_a+1} - \frac{1}{k_b+1}\right) > 0 \\ \frac{\partial P_a}{\partial\,(c_b - c_a)} &= 1 - \frac{r}{(k_a+1)^2} + \frac{rk_ak_b}{(k_b+1)^2} > \frac{rk_ak_b}{(k_b+1)^2}. \end{aligned} \tag{7}$$

As shown in the above expressions, there is a positive correlation between the average contribution c_b of neighbors and the payoff of agent a. Thus, a has a higher possibility of changing into a more cooperative allocation strategy. If the evolutionary rule is generous-first, the agent in the lower-payoff layer is very sensitive to the defective behaviors because only more defective behaviors (much lower cost than the average level in the layer) can bring higher payoff. Consequently, the agent tends to adopt the defective strategy and cooperation behaviors gradually disappear.

However, greedy-first agents can update the allocation strategy based on a comparatively higher level of average contributions, and it helps to improve the maintenance of cooperation in multiplex network. Therefore, greedy-first mechanism is better than generous-first mechanism for the evolutionary dynamics.

According to all the above analyses, cooperative behaviors can be retained in the scale-free layer, which means that a scale-free layer promotes cooperation in multiplex networks. Based on the greedy-first mechanism, agents prefer to update the allocation strategy in the scale-free layer if the conjoint layer is homogeneous. As a result, the average level of contributions in the scale-free layer can be kept; and cooperative behaviors in the other layer can also exist because agents do not directly update the strategies although the average level of contributions is lower than the one in the scale-free layer. This process helps to avoid the repeated defection in the lower-payoff (homogeneous) layer because defective strategies are easily imitated. However, if the two layers of multiplex networks are both homogeneous, only the lower contribution can lead to the higher payoff. This process certainly causes the disappearance of cooperation in all the layers of multiplex networks.

4. Simulation

The evolutionary dynamics in multiplex networks has been simulated on a computer. Multiplex networks are constructed based on Erdős–Rényi model [29], small-world model [30] and scale-free model [31]. The scale-free layer (SF) leads to the degree diversity in multiplex networks. The Erdős–Rényi random layer (ER) and small-world layer (SW) represent the homogeneous networks. There are 10 000 agents, and the average degree in each layer is 10. In the small-world layer, the probability of interpolating between regular lattices is 0.1. In the scale-free layer, the number of seed nodes is 50 with an average of five edges, and each subsequent node is added with five edges.

Two factors are used to roughly describe the level of cooperation in multiplex networks: the fraction of defective agents, who contribute nothing in the duplex layers; and the fraction of agents, who contribute all of resources. The corresponding results are drawn in line graphs. The details of evolutionary dynamics are described by the fraction of agents with different allocation strategies and shown in 3D-surface graphs. The results are averaged over 100 realizations. Each realization contains 10 million time steps.

4.1. Diversity promotes cooperation

In this section, our aim is to show the positive effect of degree diversity on evolutionary cooperation in multiplex networks. There are three types of multiplex networks which are denoted by SF+ER, SF+SW and SW+ER according to the structures of the duplex layers. The enhancement factor r is varied from 0.5 to 10. Meanwhile, all agents are greedy-first.

As shown in Fig. 3(a), nearly all agents in SW+ER multiplex networks are defective when the enhancement factor is less than 7.5. Meanwhile, the fraction of cooperative agents who contribute all the resources into SW+ER multiplex networks is much less than the one in SF+ER or SF+SW multiplex networks. It means that cooperative behaviors almost disappear in SW+ER multiplex networks when enhancement factor is less than 7.5. The average degrees in the scale-free, small world and random layers are all near 10. However, the scale-free layer can greatly improve the cooperative level in multiplex networks.

Meanwhile, if the enhancement factor is larger than 7.5, the fraction of defective agents in SW+ER multiplex networks reduces rapidly, and the fraction of cooperative agents who contribute all the resources largely increases in the three types of multiplex networks. The increase of the enhancement factor greatly improves the payoffs of agents and reduces the relative differences of payoffs between cooperative and defective agents. It finally facilitates the existence of cooperative behaviors. However, as shown in Fig. 3(b), the small-world layer can induce more cooperative agents in multiplex networks by comparing the fraction of cooperative agents in SF+ER multiplex networks and the one in SF+SW or SW+ER networks. Network reciprocity can explain the advantage of the small world layer [23]. The local clustering coefficient of the small world layer is much larger than the one of other layers. When the high value of the enhancement factor facilitates the cooperative behaviors, agents in the small world layer have more probability to imitate cooperative neighbors because of the local reinforcement [32].

4.2. How to conjoin two high-diversity layers

In order to promote cooperative behaviors in multiplex networks, what is the best method to conjoin two scale-free layers? This section tries to provide some basic suggestions to solve this problem. There are two types of multiplex networks: Type I and Type II. In Type I multiplex networks, the same set of agents is selected as seed nodes, and hubs are conjoint in the duplex scale-free layers. In Type II multiplex networks, the nodes, whose degrees are the least in the first layer, are selected as seed nodes in the other layer; and hub nodes are not conjoint in the duplex layers. Thus, the degrees of conjoint nodes in Type II multiplex networks are more diverse than the one in Type I multiplex networks. The enhancement factor r is varied from 0.5 to 10. The results are shown in Fig. 4.

It can be found that Type II multiplex networks are generally better than Type I multiplex networks. If the enhancement factor is less than 2, the fraction of defective agents in Type II multiplex networks is larger than the one in Type I multiplex networks. If the enhancement factor is larger than 2, there are more agents who contribute all the resources in Type II multiplex networks. In fact, when the agent represents the conjoint hubs in the duplex layers, the percentage of allocated resources in one layer is certainly less than the one in the other layer which can lead to the spread of less cooperative behaviors (even defection) in one layer. Therefore, degree diversity between conjoint nodes is also important to improve prospects for cooperation in multiplex networks.

4.3. Greed is better

The main object of this section is to show that greedy-first mechanism is better than generous-first mechanism. Multiplex networks consist of a scale-free layer and an Erdős–Rényi random layer. The enhancement factor r is varied from 0.5 to 10.

In Fig. 5(a), it can be found that there are more defective agents with generous-first mechanism. As the enhancement factor r is varied from 0.5 to 10, the fractions of defective agents with the two mechanisms both decrease. When the enhancement factor is lower than 4.5, all the agents in multiplex networks are defective if agents are generous-first. Correspondingly, most of agents are not defective if evolutionary rule is greedy-first and the enhancement factor approaches zero. Therefore, the critical threshold of the enhancement factor with generous-first mechanism, below which cooperative behaviors disappear in multiplex networks, is much larger than the one with greedy-first mechanism.

The details of evolutionary dynamics with generous-first mechanism are present in Figs. 6 and 7. The enhancement factors are 4 and 8, and the initial distribution of agents with different allocation strategies is drawn in Fig. 5(b). In Fig. 6, cooperative behaviors quickly disappear in the duplex layers. At the time $t = 1$ million, only a very small fraction of agents allocate very limited amount of resources ($c_i^{L_1} \approx 0$) in the scale-free layers, and all agents are defective at the time $t = 10$ million. In Fig. 7, it can be found that agents synchronously reduce the percentages of allocated resources in the duplex layers. Finally, at the time $t = 10$ million, most agents are partially cooperative in the scale-free layer, and only very few agents contribute more than half of the resources.

The details of evolutionary dynamics with greedy-first mechanism are shown in Fig. 8. The enhancement factors are 4 and 8, and only two snapshots are selected to save the space. It can be found that most agents are cooperative in the scale-free layer and there are still many agents who allocate part of resources in the random layer. Therefore, it can be concluded that greedy-first mechanism is better than generous-first mechanism and can greatly favor cooperative behaviors in multiplex networks.

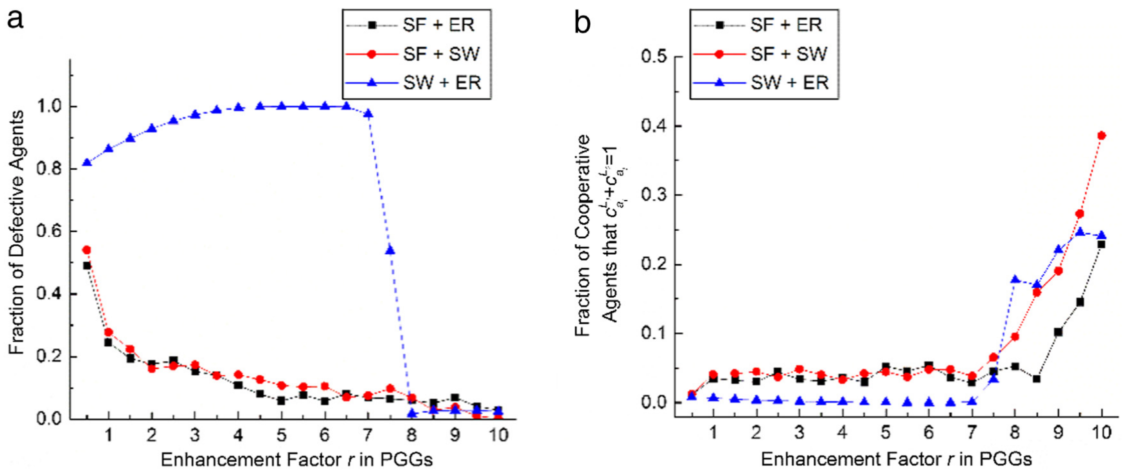

Fig. 3. Fractions of defective and cooperative agents in SF+ER, SF+SW and SW+ER multiplex networks.

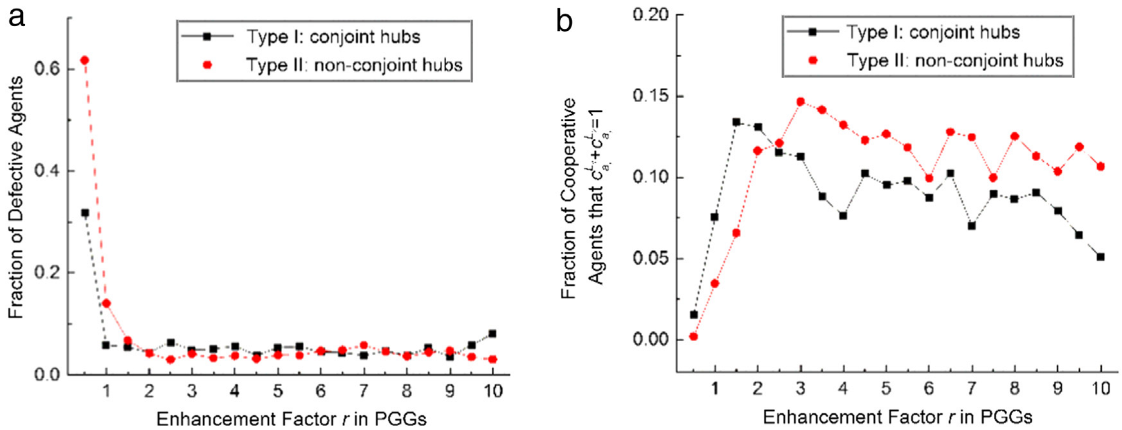

Fig. 4. Fractions of defective and cooperative agents in multiplex networks with two scale-free layers.

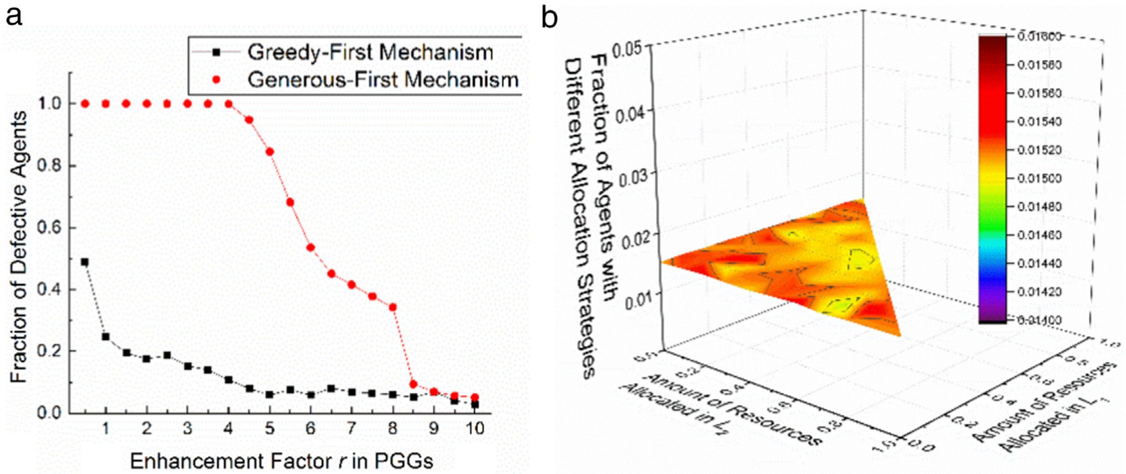

Fig. 5. (a) The final fractions of defective agents with greedy-first and generous-first mechanisms. (b) The initial distribution of agents with different allocation strategies.

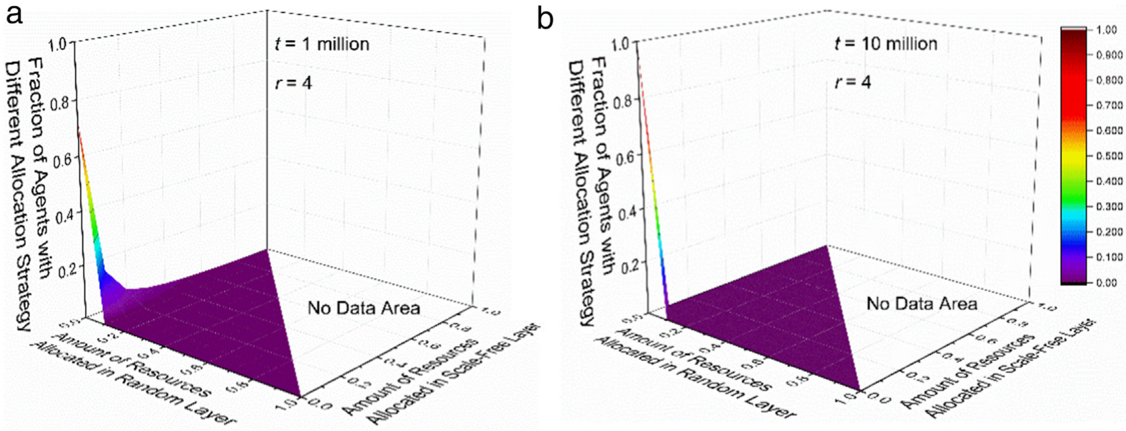

Fig. 6. The details of evolutionary dynamics with generous-first mechanism, $r = 4$. (a) $t = 1$ million; (b) $t = 10$ million.

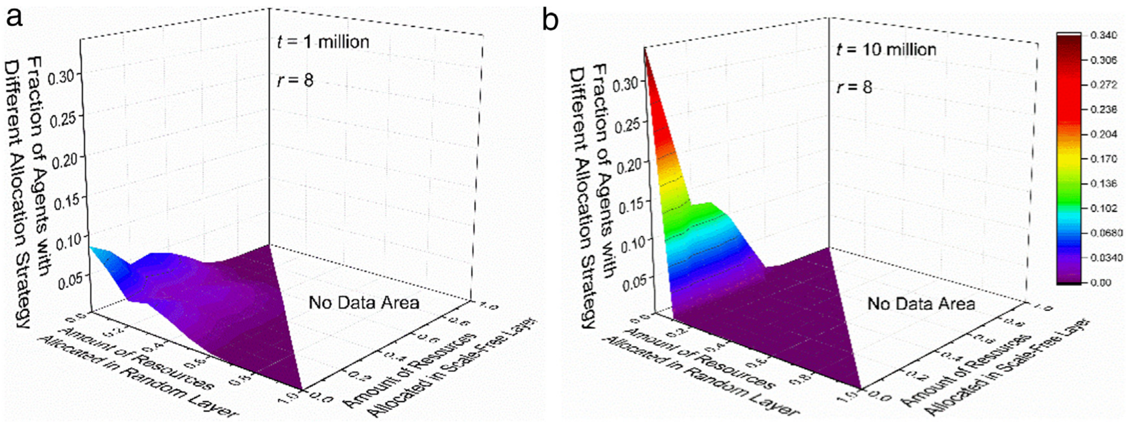

Fig. 7. The details of evolutionary dynamics with generous-first mechanism, $r = 8$. (a) $t = 1$ million; (b) $t = 10$ million.

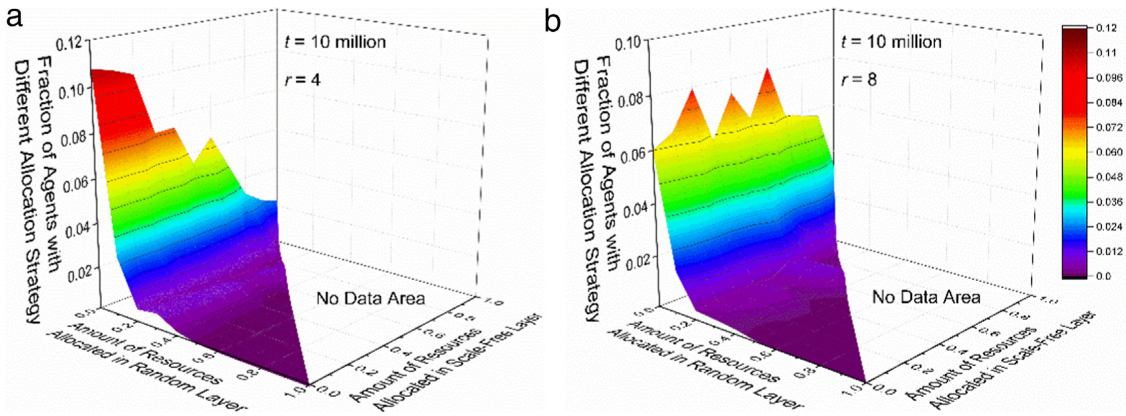

Fig. 8. The details of evolutionary dynamics with greedy-first mechanism. (a) $r = 4$, $t = 10$ million; (b) $r = 8$, $t = 10$ million.

5. Discussion and conclusion

In this paper, we try to explain the maintenance of cooperation in evolutionary dynamics with limited and partible resources in multiplex networks. It is found that degree diversity and greedy-first mechanism can defeat the temptation of defective behaviors and avoid the extremely biased cooperation in a certain layer.

From two perspectives in multiplex networks, we further extend the positive effects of degree diversity (also known as social diversity or network heterogeneity) on the maintenance of cooperation [24]. First, even if hub nodes only exist in one layer, the emergence of cooperation in multiple layers can be promoted. It is because hub nodes can utilize the contributions of numerous neighbors to cover the high costs of cooperative behaviors and then cooperation can spread among other nodes. Second, degree diversity between conjoint nodes can also facilitate cooperative behaviors in multiplex networks. This type of degree diversity helps to avoid the conflict of allocating resources into multiple layers, and hub agents can be cooperative in different layers. These conclusions provide essential principles and additional suggestions to construct multiplex networks to favor cooperative behaviors besides the cluster and community structures based on network reciprocity [23].

Greedy-first mechanism helps to keep the cooperative behaviors in the higher-payoff layer and greatly reduce the probabilities of agents to imitate the defective strategies in the lower-payoff layer. It indicates that cooperation in evolutionary dynamics is not contradictory to the human instinct of searching higher benefit when individuals are in the face of multiple choices. This instinct may cause that individuals pay close attention to different types of interaction in social life and partly ignore some unimportant interactions because of the low payoffs. These spontaneous behaviors to maintain the high payoffs in different types of interactions may finally lead to the emergence of cooperation in human social systems. In fact, greedy-first mechanism cannot guarantee that all the agents entirely contribute individuals' resources into multiple layers. In real life, few rules can lead to the selfless crowd. The feasible method may be exploiting the instinct of searching higher benefit to promote cooperative behaviors.

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