On Self-adjustment of Social Conventions to Small Perturbations [*]

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We present a model for self-adjustment of social conventions to small perturbations, and investigate how perturbations can influence the convergence of social convention in different situations. The experimental results show that the sensitivity of social conventions is determined by not only the perturbations themselves but also the agent adjustment functions for the perturbations; and social conventions are more sensitive to the outlier agent number than to the strategy fluctuation magnitudes and localities of perturbations.

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In a multi-agent system with social conventions,[1−7] there may be some outliers who occasionally violate the conventions; and perturbations are thus brought out. In some cases, the perturbations are easy to remedy; but in other cases, the perturbations will influence the fundamental characteristics of system, and may have large, unpredictable effects.[8]

In this Letter, we provide a new concept: the sensitivity of social conventions to small perturbations. If a small perturbation may be remedied well in a limited scope and quickly, we can say that the sensitivity of social conventions to such perturbation is low; but in other cases, the small perturbation may pervade over the whole system, and even brings out the emergence of a new convention; then we can say that the sensitivity of social conventions to such perturbation is high.

Definition 1. Let n be the number of agents, the multi-agent coordination is a tuple $\langle A, S, U \rangle$, where $A = \{a_1, a_2, \cdots, a_n\}$ is a set of agents, $S = \{S_1, S_2, \cdots, S_n\}$, and S_i denotes the set of social strategies available to agent a_i, $U : S_1 \times S_2 \times \cdots \times S_n \to \mathbb{R}$ is the global utility function of the multi-agent system.

Definition 2. Given the multi-agent coordination $\langle A, S, U \rangle$, a social law is the restriction of S_1 to $S_1^* \subseteq S_1$, S_2 to $S_2^* \subseteq S_2 \cdots S_n$ to $S_n^* \subseteq S_n$, so as to $\forall S_i, U(S_1^*, S_2^*, \cdots, S_n^*) \geq U(S_1 \S_1^*, S_2\ S_2^*, \cdots, S_n\ S_n^*)$.

Definition 3. According to Ref. [3], a social law that restricts agents' behaviour to one particular strategy is called a social convention; therefore, we can also simply use such particular strategy to represent the social convention.

Definition 4. Referring to Ref. [2], now we formalize our notion of convention convergence in multi-agent systems. Let S be the set of all strategies in the system, we denote by likeness (σ, ε) the set of agents that choose any strategies in strategy set S', which satisfies the following situation:

$$(S' \subseteq S) \wedge (\forall s \in S' \Rightarrow W(s, \sigma) \leq \varepsilon), \qquad (1)$$

where $W(s, \sigma)$ denotes the difference between strategy s and σ, ε denotes a predefined tolerance value. Obviously, the strategy of social convention c should satisfy

$$c = \arg \max_{\sigma \in S} |\text{likeness}(\sigma, \varepsilon)|, \qquad (2)$$

where $|\text{likeness}(\sigma, \varepsilon)|$ denotes the number of agents in likeness(σ, ε). The convergence of convention c is defined as

$$conv(c, \varepsilon) = \frac{|\text{likeness}(c, \varepsilon)|}{|A|}, \qquad (3)$$

where $|A|$ denotes the number of agents in the whole system.

Definition 5. A perturbation is the change of an agent or some agents' strategies which are deviated from or against the current social convention; those agents that initially produce the perturbation are called the outlier agents.

Definition 6. Let $conv(c, \varepsilon)$ be the convergence of social convention c before perturbation Θ take places, $conv'(c, \varepsilon)$ be the one after Θ take places, then our measure of 'sensitivity of social convention c to perturbation Θ' is

$$\Psi_c(\Theta) = 1 - \frac{conv'(c, \varepsilon)}{conv(c, \varepsilon)}. \qquad (4)$$

Local diffusion effects of perturbations. In reality, each agent interacts always with a small set of local 'neighbours', and individuals will adjust their behaviour over time by myopically acclimatizing themselves to their local neighbours.[9−11] Such a process will continue until the impact of the perturbation is decayed or spreads over the whole system.

Figure 1(a) is an example for the diffusion of

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agent perturbation. If the strategy of agent a violates the convention, then the agents within the local region $\{b, c, d\}$ will adjust their strategies to acclimatize themselves to agent a. If the strategy of b is changed, the agents within the local region $\{c, d, e\}$ may be influenced. However, the strategies of agents c and d have already been adjusted before, then, how can we deal with it? Now we give a presumption as follows:

Presumption 1. If an agent's strategy is adjusted in a perturbation diffusion process, such adjustment will keep to be fixed during the whole process of that perturbation.

Therefore, when the strategy of b is changed, the strategies of c and d keep to be fixed, and the strategy of e will be adjusted. If the strategy of e is changed, the strategies of b and c keep to be fixed, and the strategy of f will be adjusted. The diffusion process of the perturbation brought by a can be denoted by a directed graph, called the diffusion topology, as shown in Fig. 1(b).
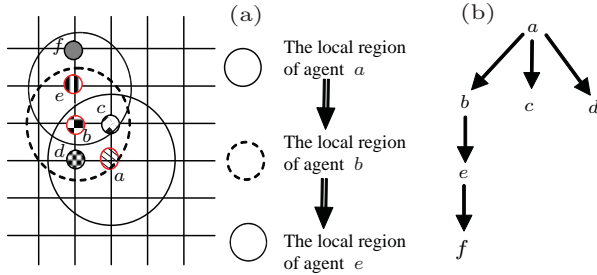

Fig. 1. An example of the perturbation diffusion process. Let the distance of each lattice be 1 and the local interaction radius be $\sqrt{2}$. (a) Diffusion process with agent a being the source of perturbation. (b) Diffusion topology.

Algorithm for the diffusion process of perturbations. Let the set of agents be A, the set of agents that originally produce the perturbation be A', the diffusion process of perturbation can be explained as algorithm 1, i.e. (1) set the tags for all agents in A to -1 initially; (2) set the tags for all agents in A' to 1; (3) create queue (Q); (4) for $\forall a \in A'$, insert (Q, a), (5) while (not empty (Q)) do: (5.1) a = out Queue (Q); (5.2) for $\forall b \in L_a$: if the tag of agent b is -1, then: (5.2.1) adjust the strategy of agent b according to b's strategy adjustment function; (5.2.2) set the tag of b to 1; (5.2.3) if the strategy of b is changed, then: (5.2.3.1) insert(Q, b); (6) end.

While the strategy of an agent is changed, its local neighbours should adjust their strategies to avoid collision, which is called minimum local conflicts.[12−14] Now, based on such a rule, we design four adjustment functions for the perturbations to reduce the conflicts. In the following sections, L_i denotes the local region of agent i.

(A) Simple Inclination to the Supreme Agent in Local Region.

Definition 8. Social ranking of agent i can be a function $p_i \rightarrow [0, \delta]$, where δ is a natural number. If $p_i > p_j$, then the social rank of agent i is superior to the one of j.

Now we design the strategy adjustment criterion of the simple inclination to the supreme agent in local region as follows.

$$j = \arg\max_{m \in (L_i \cup \{i\})} p_m, \quad s_i(t+1) = s_j(t), \quad (5)$$

where j denotes the supreme agent in the local region of agent i, $s_j(t)$ denotes the social strategy of agent j at time t.

(B) Simple Majority in Local Region.

A strategy becomes more dominant as the number of adopters increases. Therefore, each agent will change to an alternative strategy if it observes more adopters of such strategy in the local region.[8] The agents that share the same social strategy are called the overlay group of the strategy. Let $G(s)$ represents the overlay group of social strategy s, we have

$$G(s) = \{u | \text{agent } u \text{ adopts the social strategy } s\}.$$

Therefore, we can design the strategy adjustment criterion as follows:

$$s^* = \arg\max_{\substack{s \in \bigcup_{j \in (L_i \cup \{i\})} \{s_j(t)\}}} \big| G(s) \cap (\{i\} \cup L_i) \big|,$$
$$s_i(t+1) = s^*, \quad (6)$$

where s^* denotes the social strategy adopted by the majority in the local region of agent i.

(C) Simple Average in Local Region.

When many agents operate concurrently in the agent system, the agents will incline to adopt an identical average social strategy which can make the system more unified. Therefore, we design the strategy adjustment criterion of simple average in local region as follows:

$$s_i(t+1) = \underset{(j \in L_i \cup \{i\})}{\text{average}} (s_j(t)). \quad (7)$$

(D) Local Weighted Convergence Inclination.

The impact force from an agent in the local region to agent i is determined both by such agent's rank and its distance to agent i. Obviously, agent i will also influence itself. Therefore, we define the collective impact force on agent i by itself and other agents in its local region as

$$IF = \sum_{j \in L_i} \frac{p_j / p_i}{d_{ij}} + \tau, \quad (8)$$

where the first part on the left of the equation denotes the impact force of other agents within the local region to agent i, τ denotes the impact force of agent i

to itself.

Therefore, we can design the strategy adjustment criterion of local weighted convergence inclination as follows:

$$s_i(t+1) = \sum_{j \in L_i} \left[\frac{(p_j/p_i)/d_{ij}}{IF} \cdot s_j(t) \right] + \frac{\tau}{IF} \cdot s_i(t). \quad (9)$$

(A) Case Study and Experimental Environment.

Now we use the case of a multiagent system that simulates a crowd of strangers standing on a playground. In our case, the social strategy of an agent is its direction. Let n be the number of agents, we can use an array to denote the social strategies of agents. Here $s_i \to \{1, \cdots, 8\}$, $1 \le i \le n$, represents social strategy (i.e., the standing direction) of agent i.
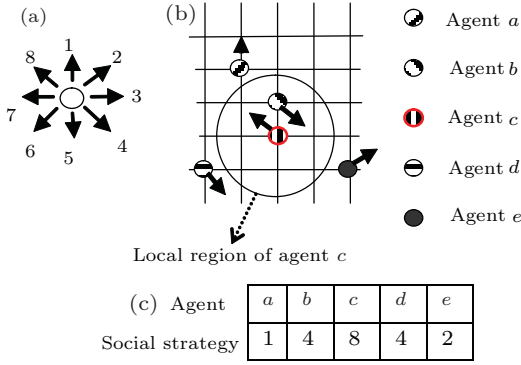

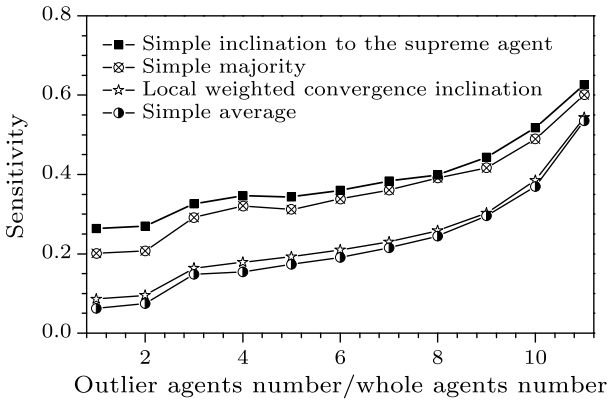**Fig. 2.** The case of an agent system and its social strategies.

Fig. 3. Test results for varying numbers of outlier agents.

Local region in our case: let the position of agent i be (x_i, y_i) and the distance of each lattice be 1 in our case, then the local interaction diffusion group of agent i is composed of the agents that locate on the place of (x, y) which satisfies

$$L_i = \{j | d(i,j) \le \sqrt{2}\}, \ (x_i - 1) \le x \le (x_i + 1),$$
$$(y_i - 1) \le y \le (y_i + 1). \quad (10)$$

Here we assume that there already exists a social convention in the system, i.e., most or all the agents

have already adopted a particular strategy (standing with the same direction). Now if one or some agents change their standing directions randomly, we will test how the perturbation will influence the convergence of the existing social convention in varying perturbations as well as different adjustment functions.

(B) Test Results and Analyses.

(1) Varying outlier agent numbers in perturbation. The results are seen in Fig. 3. If the number of outlier agents increases, the convergence of the existing convention will decrease accordingly. Therefore, the sensitivity of social convention to perturbation varies directly as the number of outlier agents in the perturbation.

(2) Varying strategy fluctuation magnitudes in perturbation. First, we change the strategies of the outlier agents for a little, and then we will increase the fluctuation magnitudes of outlier agent strategies step by step. The results are seen in Fig. 4. The results are illustrated in Fig. 4. The fluctuation magnitudes of outlier agent strategies have not obvious effects on the sensitivity of convention to the perturbations.
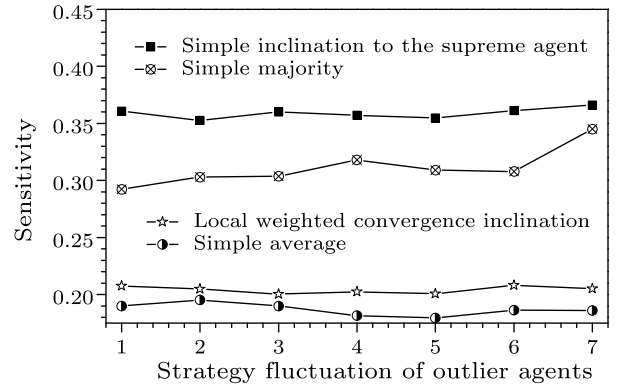

Fig. 4. Test results for varying strategy fluctuation magnitudes of outlier agents.
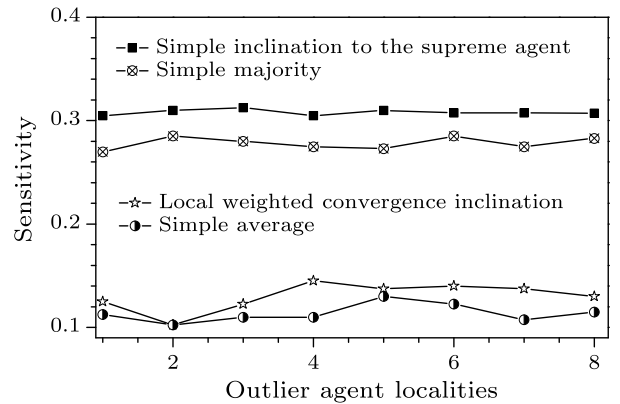

Fig. 5. Test results for varying outlier agent localities in perturbations.

(3) Varying outlier agent localities in perturbations. To consider the varying outlier agent localities in perturbations, we can use the average distance be-

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tween outlier agents and the centre of the grid (which is denotes as d_{AC}). First we can set the outlier agents to locate the outer space of the grid, then we reduce d_{AC} step by step for several cases. The results are illustrated in Fig. 5, where the x-axis denotes the list of varying d_{AC} in decreasing order. Here the outlier agent localities have not obvious effects on the sensitivity of social convention to the perturbations.

Table 1. Test results for varying situations of multiagent systems. $|A|$ denotes the number of agents in the system, a for the perturbation proportion of 20%, b for the perturbation proportion of 50%. (I) Simple inclination to the supreme agents, (II) simple majority, (III) local weighted convergence inclination, (IV) simple average.

$|A|$		100	225	400	625	900	1225	1600
I	a	0.450	0.471	0.405	0.426	0.426	0.413	0.413
	b	0.710	0.671	0.733	0.664	0.738	0.656	0.723
II	a	0.360	0.396	0.408	0.382	0.378	0.376	0.378
	b	0.580	0.597	0.600	0.610	0.613	0.605	0.608
III	a	0.220	0.280	0.265	0.245	0.261	0.277	0.260
	b	0.590	0.551	0.540	0.536	0.540	0.537	0.545
IV	a	0.230	0.244	0.235	0.237	0.246	0.248	0.243
	b	0.530	0.551	0.520	0.534	0.528	0.527	0.545
$|A|$		2025	2500	3025	3600	4900	6400	10000
I	a	0.426	0.422	0.425	0.417	0.411	0.416	0.418
	b	0.663	0.727	0.667	0.727	0.736	0.726	0.726
II	a	0.394	0.384	0.388	0.382	0.379	0.390	0.379
	b	0.603	0.609	0.604	0.615	0.603	0.609	0.610
III	a	0.253	0.258	0.256	0.261	0.260	0.260	0.261
	b	0.540	0.558	0.541	0.554	0.554	0.551	0.554
IV	a	0.245	0.239	0.248	0.243	0.247	0.247	0.245
	b	0.522	0.531	0.527	0.540	0.537	0.536	0.536

(4) Varying situations of multiagent systems. Here we consider the varying situations of multiagent systems, which include varying scales and distribution. In each test, we adopt two kinds of perturbation proportions (i.e. the proportion of outlier agents in the whole system), one is 20%, and the other is 50%. Now we increase the numbers of the whole multiagents step by step for varying cases, and in each case the agents are distributed randomly. The results are seen in Table 1. Here the number of the whole agents has not obvious effects on the sensitivity of convention to the perturbations; but the perturbation proportion has obvious effects on the sensitivity of convention to the perturbations. For the sensitivities, the four agent strategy adjustment functions can be listed in descending order: simple inclination to the supreme agent, simple majority, local weighted convergence inclination, simple average.

(6) Analyses and summary for the experimental results. The convention is more sensitive to the outlier agent number than to the outlier agent localities and strategy fluctuation magnitudes of the perturbation; the potential reason is: our strategy adjustment functions are all locally controlled, thus the perturbation always takes effects locally and each outlier agent always influences other agents locally. If we want to remedy the perturbation as locally as possible, the adjustment function of simple average in local region can remedy the perturbations better than other three adjustment functions, since it can get the unification easily.

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